A Gentle Introduction to Nomographs

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Given three points, $(x_1, y_1), (x_2, y_2), (x_3, y_3)$, and told that they are collinear meaning they all fall on the same straight line the following relationship holds:

$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_3}{x_2 - x_3} \tag{1}$$

This is simply the statement the slopes for the two pairs of points are equal. Now applying some basic algebra:

$$(x_2 - x_3)(y_1 - y_2) = (x_1 - x_2)(y_2 - y_3) \tag{2}$$

$$x_2(y_1 - y_2) - x_3(y_1 - y_2) = x_1(y_2 - y_3) - x_2(y_2 - y_3) \tag{3}$$

Noting that the term (x_2, y_2) appears on both sides, cancels out the, following expression is derived.

$$x_1(y_2 - y_3) - x_2(y_2 - y_3) - x_3(y_2 - y_3) = 0 \tag{4}$$

The response to all this may well be "So what". But recall some basic linear algebra, in particular the definition of the determinant, the above expression can be recast as:

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \tag{5}$$

We now have expression in the form of a determinant which holds for the case of three points which are collinear. Now consider the case where the individual x,y pairs can be expressed in the form of parametric equations:

$$x_1 = f_1(\alpha) y_1 = g_1(\alpha) \tag{6}$$

$$x_2 = f_1(\beta) y_2 = g_1(\beta) \tag{7}$$

$$x_3 = f_3(\gamma) y_3 = g_3(\gamma) \tag{8}$$

Consider the general case of an equation involving three variables which can be expressed in the form of a determinant:

$$\begin{vmatrix} f_1(\alpha) & g_1(\alpha) & h_1(\alpha) \\ f_2(\beta) & g_2(\beta) & h_2(\beta) \\ f_3(\gamma) & g_3(\gamma) & h_3(\gamma) \end{vmatrix} = f_1(\alpha)g_2(\beta)h_3(\gamma) + f_3(\gamma)g_1(\alpha)h_2(\beta) + f_2(\beta)g_3(\gamma)h_1(\alpha) - f_3(\gamma)g_2(\beta)h_1(\alpha) - f_1(\alpha)g_3(\gamma)h_2(\beta) - f_2(\beta)g_1(\alpha)h_3(\gamma) \tag{9}$$

Similar in spirit to separation of variable technique in solving differential equations, the first step in nomography is to expressing the 3 variable equation in form 9. Once this has been accomplished using the algebraic rules of determinants to transfrom into form 5.